Flexural analysis of laminated composite and sandwich beams using a four-unknown shear and normal deformation theory

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Abstract

This paper presents flexural analysis of composite and sandwich beams using a quasi-3D theory, which considers simultaneously three effects such as normal and shear deformation as well as anisotropy coupling. The axial and transverse displacements are assumed to be cubic and parabolic variation through the beam depth. In order to solve problem, two-node C^1 beam elements with six degrees of freedom per node are developed. Numerical examples are carried out and the results are compared with those available in literature to validate the accuracy of the present theory. The effects of fibre angle, lay-up and span-to-height ratio on displacements and stresses are studied. Some new results, which can be useful for future references, are also given.

Keywords: Composite and sandwich beams; quasi-3D theory; normal and shear deformation ; finite element method.

1. Introduction

Due to attractive properties in strength, stiffness, and lightness, laminated composite beams become popular and have been used in differerent areas such as aerospace, mechanical and structural engineering, etc. Various beam theories have been developed for analysis of their structural behaviour. A review of these theories can be found in recent work of Nguyen et al. [1]. As noted in their review, higher-order beam theories (HBTs) are considered to be one of the most popular ones since they predict more accurate than classical beam theory (CBT) and first-order beam theory (FBT) especially for thick beams. By using HBTs, there are many studies have been done to study static analysis of

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composite beams and only some of them ([2–10]) are referenced here. It should be noted that in these above papers, only shear deformation effect is included, while normal deformation effect is ignored. As discussed in Carrera et al. [11], both effects become significant for thick beams. In order to include them, the axial and transverse displacements of beams are assumed to be higher-order variation through the depth. These types of theories are namely quasi-3D ones, which are developed by many authors for static analysis of composite beams. Carrera et al. [11] proposed a novel unified approach called Carrera Unified Formulation (CUF), which is hierarchical formulation and considers the order of model as a free parameter. The most novel features of this formulation is that various beam theories such as CBT, FBT, HBT and quasi-3D theory, can be obtained without ad hoc formulations. This formulation was applied successfully to bending problem of composite beams [12–16]. Zenkour [17] derived Navier solutions to investigate deflections and stresses of cross-ply composite beams. Subramanian [18] developed two-node C^1 beam elements with eight degrees of freedom (DOFs) per node to study flexural behaviour of simply-supported symmetrically-laminated composite beams. Kant et al. [19] derived semi-analytical model for bending behaviour of cross-ply composite and sandwich beams based on solution of two-point boundary value problem governed by ordinary differential equations. Pawar et al. [20] derived Navier solution for static analysis of cross-ply composite and sandwich beams. Mantari and Canales ([21, 22]) proposed a hybrid type quasi-3D HBT for the bending analysis of cross-ply composite beams. It should be noted that in above studies ([17]-[22]), only bending behaviour of cross-ply or symmetric composite beams is considered. This problem for laminated composite beams with arbitrary lay-ups is not well-investigated and there is a need for further studies.

This work aims to study flexural analysis of composite beams with arbitrary lay-up and sandwich beams using a four-unknown shear and normal deformation theory. The axial and transverse displacements are assumed to be cubic and parabolic variation through the beam depth. As a results, three effects such as normal and shear deformation as well as anisotropy coupling are simultaneously taken into account. Two-node C^1 beam elements with six DOFs/node are developed to calculate displacements and stresses. Numerical results are compared with those predicted by other theories to investigate the effects of normal and shear deformation as well as anisotropy coupling on bending behaviour of composite and sandwich beams.

2. Theoretical formulation

2.1. Kinematics

A laminated composite beam, which is made of many plies of orthotropic materials in different orientations with respect to the x-axis, is considered (Fig. 1). The axial and transverse displacement variations are assumed to be cubic and quadratic functions of the depth [23–26]:

$$U(x,z,t) = u(x,t) - z\frac{\partial w_b(x,t)}{\partial x} - \frac{4z^3}{3h^2}\frac{\partial w_s(x,t)}{\partial x} = u(x,t) - zw_b'(x,t) - f(z)w_s'(x,t) \tag{1a}$$

$$W(x,z,t) = w_b(x,t) + w_s(x,t) + (1 - \frac{4z^2}{h^2})w_z(x,t) = w_b(x,t) + w_s(x,t) + g(z)w_z(x,t) \tag{1b}$$

where u, w_b, w_s and w_z are four mid-plane displacements of beam.

The non-zero strains are:

$$\epsilon_x = \frac{\partial U}{\partial x} = u' - zw_b'' - fw_s'' \tag{2a}$$

$$\epsilon_z = \frac{\partial W}{\partial z} = g'w_z \tag{2b}$$

$$\gamma_{xz} = \frac{\partial W}{\partial x} + \frac{\partial U}{\partial z} = g(w_s' + w_z') \tag{2c}$$

2.2. Variational Formulation

The variation of the strain energy can be stated as:

$$\begin{aligned}\delta\mathcal{U} &= \int_0^l \int_0^b \left[\int_{-h/2}^{h/2} (\sigma_x\delta\epsilon_x + \sigma_{xz}\delta\gamma_{xz} + \sigma_z g'\delta w_z)dz\right]dydx \\ &= \int_0^l \left[N_x\delta u' - M_x^b\delta w_b'' - M_x^s\delta w_s'' + Q_{xz}(\delta w_s' + \delta w_z') + R_z\delta w_z\right])dx\end{aligned} \tag{3}$$

where $N_x, M_x^b, M_x^s, Q_{xz}$ and R_z are the stress resultants, defined as:

$$N_x = \int_{-h/2}^{h/2} \sigma_x b dz \tag{4a}$$

$$M_x^b = \int_{-h/2}^{h/2} \sigma_x z b dz \tag{4b}$$

$$M_x^s = \int_{-h/2}^{h/2} \sigma_x f b dz \tag{4c}$$

$$Q_{xz} = \int_{-h/2}^{h/2} \sigma_{xz} g b dz \tag{4d}$$

$$R_z = \int_{-h/2}^{h/2} \sigma_z g' b dz \tag{4e}$$

The variation of the potential energy under a transverse load q can be written as

$$\delta\mathcal{V} = -\int_0^l q(\delta w_b + \delta w_s + g\delta w_z)dx \tag{5}$$

By using the principle of total potential energy, the following weak statement is obtained:

$$0 = \int_0^l \Big[N_x\delta u' - M_x^b\delta w_b'' - M_x^s\delta w_s'' + Q_{xz}(\delta w_s' + \delta w_z') + R_z\delta w_z - q(\delta w_b + \delta w_s + g\delta w_z)\Big]dx \tag{6}$$

2.3. Constitutive Equations

The constitutive equation is reduced from the 3D stress-strain relationship of a k^{th} orthotropic lamina by setting the stresses σ_y, σ_{yz} and σ_{xy} equal to zero:

$$\begin{Bmatrix} \sigma_x \\ \sigma_z \\ \sigma_{xz} \end{Bmatrix}^k = \begin{bmatrix} \overline{Q}^*_{11} & \overline{Q}^*_{13} & 0 \\ \overline{Q}^*_{13} & \overline{Q}^*_{33} & 0 \\ 0 & 0 & \overline{Q}^*_{55} \end{bmatrix}^k \begin{Bmatrix} \epsilon_x \\ \epsilon_z \\ \gamma_{xz} \end{Bmatrix} \tag{7}$$

where

$$\overline{Q}^*_{11} = \overline{Q}_{11} + \frac{\overline{Q}^2_{16}\overline{Q}_{22} - 2\overline{Q}_{12}\overline{Q}_{16}\overline{Q}_{26} + \overline{Q}^2_{12}\overline{Q}_{66}}{\overline{Q}^2_{26} - \overline{Q}_{22}\overline{Q}_{66}} \tag{8a}$$

$$\overline{Q}^*_{13} = \overline{Q}_{13} + \frac{\overline{Q}_{16}\overline{Q}_{22}\overline{Q}_{36} + \overline{Q}_{12}\overline{Q}_{23}\overline{Q}_{66} - \overline{Q}_{16}\overline{Q}_{23}\overline{Q}_{26} - \overline{Q}_{12}\overline{Q}_{26}\overline{Q}_{36}}{\overline{Q}^2_{26} - \overline{Q}_{22}\overline{Q}_{66}} \tag{8b}$$

$$\overline{Q}^*_{33} = \overline{Q}_{33} + \frac{\overline{Q}^2_{36}\overline{Q}_{22} - 2\overline{Q}_{23}\overline{Q}_{26}\overline{Q}_{36} + \overline{Q}^2_{23}\overline{Q}_{66}}{\overline{Q}^2_{26} - \overline{Q}_{22}\overline{Q}_{66}} \tag{8c}$$

$$\overline{Q}^*_{55} = \overline{Q}_{55} - \frac{\overline{Q}^2_{45}}{\overline{Q}_{44}} \tag{8d}$$

where $\overline{Q}_{ij}$ are the transformed reduced stiffness constants [27].

If the shear strain γ_{xy} is also ignored or for unidirectional and cross-ply lay-ups, Eqs. (8a)-(8c) become:

$$\overline{Q}^*_{11} = \overline{Q}_{11} - \frac{\overline{Q}^2_{12}}{\overline{Q}_{22}} \tag{9a}$$

$$\overline{Q}^*_{13} = \overline{Q}_{13} - \frac{\overline{Q}_{12}\overline{Q}_{23}}{\overline{Q}_{22}} \tag{9b}$$

$$\overline{Q}^*_{33} = \overline{Q}_{33} - \frac{\overline{Q}^2_{23}}{\overline{Q}_{22}} \tag{9c}$$

The stress resultants can be expressed in terms of displacements by substituting Eqs. (7) and (2) into Eq. (4):

$$\begin{Bmatrix} N_x \\ M^b_x \\ M^s_x \\ R_z \\ Q_{xz} \end{Bmatrix} = \begin{bmatrix} A & B & B_s & X & 0 \\ & D & D_s & Y & 0 \\ & & H & Y_s & 0 \\ & & & Z & 0 \\ sym. & & & & A_s \end{bmatrix} \begin{Bmatrix} u' \\ -w''_b \\ -w''_s \\ w_z \\ w'_s + w'_z \end{Bmatrix} \tag{10}$$

where

$$
\begin{aligned}
(A, B, B_s, D, D_s, H) &= \int_{-h/2}^{h/2} \overline{Q}^*_{11}(1, z, f, z^2, fz, f^2, g'^2)bdz && (11a)\\
(X, Y, Y_s) &= \int_{-h/2}^{h/2} \overline{Q}^*_{13} g'(1, z, f)bdz && (11b)\\
A_s &= \int_{-h/2}^{h/2} \overline{Q}^*_{55} g^2 bdz && (11c)\\
Z &= \int_{-h/2}^{h/2} \overline{Q}^*_{33} g'^2 bdz && (11d)
\end{aligned}
$$

2.4. Governing Equations

By integrating by parts and collecting the coefficients of $\delta u, \delta w_b, \delta w_s$ and δw_z, the governing equations can be obtained :

$$
\begin{aligned}
N'_x &= 0 && (12a)\\
M^{b\prime\prime}_x + q &= 0 && (12b)\\
M^{s\prime\prime}_x + Q'_{xz} + q &= 0 && (12c)\\
Q'_{xz} - R_z + gq &= 0 && (12d)
\end{aligned}
$$

The natural boundary conditions are of the form:

$$
\begin{aligned}
\delta u &: N_x && (13a)\\
\delta w_b &: M^{b\prime}_x && (13b)\\
\delta w'_b &: M^b_x && (13c)\\
\delta w_s &: M^{s\prime}_x + Q_{xz} && (13d)\\
\delta w'_s &: M^s_x && (13e)\\
\delta w_z &: Q_{xz} && (13f)
\end{aligned}
$$

By substituting Eq. (10) into Eq. (12), the governing equations can be expressed:

$$
\begin{aligned}
Au'' - Bw'''_b - B_s w'''_s + Xw'_z &= 0 && (14a)\\
Bu''' - Dw^{iv}_b - D_s w^{iv}_s + Yw''_z + q &= 0 && (14b)\\
B_s u''' - D_s w^{iv}_b - Hw^{iv}_s + A_s w''_s + (A_s + Y_s)w''_z + q &= 0 && (14c)\\
-Xu' + Yw''_b + (A_s + Y_s)w''_s + A_s w''_z - Zw_z + gq &= 0 && (14d)
\end{aligned}
$$

3. Finite Element Formulation

A two-node C^1 beam element with six DOFs/node is developed. Linear polynomial Ψ_j is used for u and w_z and Hermite-cubic polynomial ψ_j is used for w_b and w_s. The displacements within an element are expressed as:

$$u = \sum_{j=1}^{2} u_j \Psi_j \tag{15a}$$

$$w_b = \sum_{j=1}^{4} w_{bj} \psi_j \tag{15b}$$

$$w_s = \sum_{j=1}^{4} w_{sj} \psi_j \tag{15c}$$

$$w_z = \sum_{j=1}^{2} w_{zj} \Psi_j \tag{15d}$$

Substituting Eqs. (15) into Eq. (6), the finite element model of a typical element can be expressed as:

$$\begin{bmatrix} K_{11} & K_{12} & K_{13} & K_{14} \\ & K_{22} & K_{23} & K_{24} \\ & & K_{33} & K_{34} \\ \text{sym.} & & & K_{44} \end{bmatrix} \begin{Bmatrix} u \\ w_b \\ w_s \\ w_z \end{Bmatrix} = \begin{Bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \end{Bmatrix} \tag{16a}$$

where $[K]$ is the element stiffness matrix, given by:

$$K_{ij}^{11} = \int_0^l A\Psi_i'\Psi_j' dx; \quad K_{ij}^{12} = -\int_0^l B\Psi_i'\psi_j'' dx; \tag{17a}$$

$$K_{ij}^{13} = -\int_0^l B_s\Psi_i'\psi_j'' dx; \quad K_{ij}^{14} = \int_0^l X\Psi_i'\Psi_j dx \tag{17b}$$

$$K_{ij}^{22} = \int_0^l D\psi_i''\psi_j'' dx; \quad K_{ij}^{23} = \int_0^l D_s\psi_i''\psi_j'' dx; \tag{17c}$$

$$K_{ij}^{24} = -\int_0^l Y\psi_i''\Psi_j dx \tag{17d}$$

$$K_{ij}^{33} = \int_0^l (H\psi_i''\psi_j'' + A_s\psi_i'\psi_j') dx; \tag{17e}$$

$$K_{ij}^{34} = \int_0^l (-Y_s\psi_i''\Psi_j + A_s\psi_i'\Psi_j') dx \tag{17f}$$

$$K_{ij}^{44} = \int_0^l (Z\Psi_i\Psi_j + A_s\Psi_i'\Psi_j') dx \tag{17g}$$

and $[F]$ is the element force vector, given by:

$$F_i^1 = \int_0^l \mathcal{P}_x\Psi_i dx \tag{18a}$$

$$F_i^2 = \int_0^l \mathcal{P}_z \psi_i dx \tag{18b}$$

$$F_i^3 = \int_0^l \mathcal{P}_z \psi_i dx \tag{18c}$$

$$F_i^4 = \int_0^l g\mathcal{P}_z \Psi_i dx \tag{18d}$$

4. Numerical Examples

In this section, a number of numerical examples are illustrated to verify the accuracy of the present study and investigate the displacements and stresses of composite beams with arbitrary lay-up and sandwich beams with various configurations. These beams with different lay-ups, boundary conditions (clamped-clamped, cantilever and simply-supported) and span-to-height ratios are considered. Five types of material set, which relate to each example, are given in Table 1. For convenience, the vertical displacement and stresses of beams under the uniformly distributed load q are defined below as non-dimensional terms:

$$\overline{w} = \frac{wbhE_2h^2 10^2}{qL^4} \tag{19a}$$

$$\overline{\sigma}_x = \frac{bh^2}{qL^2}\sigma_x(L/2, h/2) \tag{19b}$$

$$\overline{\sigma}_z = \frac{b}{q}\sigma_x(L/2, h/2) \tag{19c}$$

$$\overline{\sigma}_{xz} = \frac{bh}{qL}\sigma_{xz}(0, 0) \tag{19d}$$

Example 1: Symmetric and anti-symmetric cross-ply composite beams with various boundary conditions and span-to-height ratios (L/h = 5, 10 and 50) are considered. All laminate are the same thickness and material properties (MAT 1). The mid-span displacements and stresses are given in Tables 2 and 3. The present results are compared with those obtained from various authors using HBTs ([3, 7, 17]) and quasi-3D theories ([17, 21]). It can be seen that the present results agree well with previous ones for both theories. It should be noted that the results with $\epsilon_z \neq 0$ (quasi-3D) are slightly different from those without it ($\epsilon_z = 0$, HBT), especially for thick beam ($L/h = 5$) and anti-symmetric cross-ply lay-up. Distributions of normal stress, shear stress and transverse normal stress through-the-thickness are plotted in Figs. 2-4. It is from transverse normal stress in Fig. 4 that highlights the importance of normal deformation effect on bending behaviour of composite beam. As noted before, since HBTs ([3, 7]) ignore this effect, thus transverse normal stress can not be observed.

Example 2: This example is extended from previous one, symmetric $[0^\circ/\theta/0^\circ]$ and unsymmetric $[0^\circ/\theta]$ composite beams are considered. Variations of displacement and stresses respect to the fibre angle are given in Tables 4 and 5. Third-order beam theory (TBT) solutions from previous study [24] are also included to show the effect of normal deformation on beams' displacements. As the fibre angle increases, all maximum stresses and displacement increase (Figs. 2-6). It is interesting to observe that normal deformation effect depends not only on span-to-height ratio but also boundary conditions and lay-ups. For thick beam, it is more pronounced for clamped-clamped boundary conditions and unsymetric lay-up than others. It can be explained partly from anisotropic coupling terms X, Y and Y_s in Eq. (10). These terms for unsymmetric lay-up are larger than those of symmetric one. As span-to-height ratio increases (Fig. 6), normal deformation effect becomes negligible, thus the results from TBT and quasi-3D theory are the same for both lay-ups, as expected.

Example 3: Cross-ply sandwich beams ($[0^\circ/90^\circ/0^\circ]$, MAT 2) with the top and bottom face thickness h_1 and core thickness h_2, are considered. The deflection and stresses of simply-supported beam with $h_2/h_1 = 3$ and 8 are compared with Zenkour [17] in Tables 6 and 7. It can be seen that the present results coincide with previous ones. Stresses and displacement distributions through-the-thickness are plotted in Figs. 7 and 8. As expected, displacement has parabolic distribution with peak point at the mid-plane. As the thickness ratio (h_2/h_1) changes from 3 to 8, normal stress and displacement increase while shear stress decreases. Variations of displacement of simply-supported and clamped-clamped sandwich beams with respect to h_2/h_1 for various L/h are plotted in Fig. 9. It is interesting to see that as h_2/h_1 increases, their response are different, which depends on boundary conditions and thin or thick beams. For simply-supported thin beam (L/h=50), displacement monotonically increases, whereas, for thick ones, it decreases to minimum value at h_2/h_1=1 and 1.8 for L/h=10 and 5, respectively and then increases.

Example 4: The validity and accuracy of the present theory is further investigated for cross-ply sandwich beams with soft core, which are made of five layers $[0^\circ/90^\circ/\text{Core}/90^\circ/0^\circ]$ (MAT 3), are considered. The thickness of each face is $0.05h$ and of core is $0.8h$. The results are compared with those using higher order zigzag theory [28] in Table 8. It is observed that the solutions of the two approaches are in excellent agreement although there are small discrepancy in displacement for L/h=5. The results for clamped-clamped and clamped-free are also presented in Table 9. They have not been reported before and could be served as benchmark examples for future references.

Example 5: Sandwich beams with soft core made of three layers $[0^\circ/\text{Core}/0^\circ]$, which have the same thickness of core and face with example 4, are considered. Two different material sets (MAT 4 and MAT 5) are used to investigate the effect of core stiffness on their displacement and stresses.

The results are also compared with those from Kant et al. [19] and Pawar et al. [20] and Carrera et al. [29]. It should be noted that normal and shear deformation effects are included in these studies with different models. Again, excellent agreement with previous studies can be observed, especially with those from Pawar et al. [20]. As beam gets thinner, the present results agree well with those of Carrera et al. [29] for various boundary conditions.

5. Conclusions

A finite element model for flexural behaviour of laminated composite and sandwich beams using a quasi-3D theory is presented. Composite and sandwich beams with various configurations including boundary conditions, span-to-height ratio and lay-ups are considered. Numerical results are compared with those predicted by other theories to show the validity of present model. Normal deformation effect depends not only on span-to-height ratio but also boundary conditions and lay-ups. For thick beam, it is more pronounced for clamped-clamped boundary conditions and unsymetric lay-up than others. Effects of normal and shear deformation as well as anisotropy coupling should be simultaneously considered to predict accurately displacements and stresses of composite and sandwich beams under vertical loads.

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CAPTIONS OF TABLES

Table 1: Material properties of composite and sandwich beams.

Table 2: Normal and shear stresses of [0°/90°/0°] and [0°/90°] simply-supported beams under a uniformly distributed load (MAT 1).

Table 3: Normal and shear stresses of [0°/90°/0°] and [0°/90°] simply-supported beams under a uniformly distributed load (MAT 1).

Table 4: Mid-span displacements of [0°/θ/0°] and [0°/θ] beams under a uniformly distributed load.

Table 5: Normal and shear stresses of [0°/θ/0°] and [0°/θ] simply-supported beams under a uniformly distributed load (MAT 1).

Table 6: Mid-span displacements of [0°/90°/0°] ($h_2/h_1 = 3$ and $h_2/h_1 = 8$) sandwich beams under a uniformly distributed load (MAT 2).

Table 7: Normal and shear stresses of [0°/90°/0°] ($h_2/h_1 = 3$ and $h_2/h_1 = 8$) simply-supported beams under a uniformly distributed load (MAT 2).

Table 8: Displacements and stresses of [0°/90°/C/90°/0°] simply-supported beams under a uniformly distributed load (MAT 3, $\overline{u} = \frac{u(0,h/2)E_2}{hq}$, $\overline{\sigma}_{xz} = \frac{\sigma_{xz}(0,0)}{q}$, $\overline{\sigma}_x = \frac{\sigma_x(L/2,h/2)}{q}$).

Table 9: Displacements and stresses of [0°/90°/C/90°/0°] clamped-clamped and cantilever beams under a uniformly distributed load (MAT 3, $\overline{\sigma}_{xz} = \frac{\sigma_{xz}(0,0)}{q}$, $\overline{\sigma}_x = \frac{\sigma_x(L/2,h/2)}{q}$).

Table 10: Mid-span displacements of [0°/Core/0°] sandwich beams under a uniformly distributed load (MAT 4 and MAT 5).

Table 11: Normal and shear stresses of [0°/Core/0°] simply-supported sandwich beams under a uniformly distributed load (MAT 4 and 5).

Table 1: Material properties of composite and sandwich beams.

Example	Material set	Material properties
1, 2	MAT 1 [30]	$E_1/E_2 = 25, E_3 = E_2, G_{12} = G_{13} = 0.5E_2, G_{23} = 0.2E_2,$
		$\nu_{12} = \nu_{13} = \nu_{23} = 0.25$
3	MAT 2 [17]	Face layer: MAT 1
		Core layer: $E_1/E_2 = 1, E_3 = E_2, G_{12} = G_{13} = 1.5E_2, G_{23} = 0.4E_2,$
		$\nu_{12} = \nu_{13} = \nu_{23} = 0.25$
4	MAT 3 [28]	Face layer 1: MAT 1
		Face layer 2: $E_1/E_2 = 1, E_3 = E_2, G_{12} = G_{13} = G_{23} = 0.5E_2,$
		$\nu_{12} = \nu_{13} = \nu_{23} = 0.25$
		Core layer: $E_1/E_2 = 80, E_3 = E_2, G_{12} = G_{13} = G_{23} = 1.2E_2,$
		$\nu_{12} = \nu_{13} = \nu_{23} = 0.25$
5	MAT 4 [30]	Face layer: E_1 = 172.4 GPa, E_2 = 6.89 GPa, $E_3 = E_2$,
		$G_{12} = G_{13}$= 3.45 GPa, G_{23} = 1.378 GPa,
		$\nu_{12} = \nu_{13} = \nu_{23} = 0.25$
		Core layer: $E_1 = E_2$ = 0.276 GPa, E_3 = 3.45 GPa,
		G_{12} = 0.1104 GPa,$G_{23} = G_{13}$ = 0.414 GPa, $\nu_{12} = \nu_{13} = \nu_{23} = 0.25$
5	MAT 5 [31]	Face layer: E_1 = 131.1 GPa, E_2 = 6.9 GPa, $E_3 = E_2$,
		G_{12} = 3.588 GPa, G_{13} = 2.3322 GPa, G_{23} = 3.088 GPa,
		$\nu_{12} = \nu_{13} = 0.32$, $\nu_{23} = 0.49$
		Core layer: E_1 = 0.2208 MPa, E_2 =0.2001 MPa, E_3 = 0.2760 MPa,
		G_{12} = 16.56 MPa, G_{13} = 545.1 MPa, G_{23} =455.4 MPa,
		ν_{12} =0.99, $\nu_{13} = \nu_{23} = 3 \times 10^{-4}$

Table 2: Mid-span displacements of [0°/90°/0°] and [0°/90°] beams under a uniformly distributed load (MAT 1).

Theory	Reference	Symmetric ([0°/90°/0°])			Anti-symmetric ([0°/90°])		
		$L/h = 5$	10	50	$L/h = 5$	10	50
a. Cantilever beams							
HBT	Khdeir and Reddy [3]	6.8240	3.4550	2.2510	15.2790	12.3430	11.3370
($\epsilon_z = 0$)	Murthy et al. [7]	6.8360	3.4660	2.2620	15.3340	12.3980	11.3920
	Present	6.8304	3.4607	2.2568	15.3050	12.3690	11.3630
Quasi-3D	Mantari and Canales [21]	-	3.4592	-	-	12.4750	-
($\epsilon_z \neq 0$)	Present	6.8541	3.4605	2.2565	15.1540	12.2440	11.2580
b. Simply-supported beams							
HBT	Khdeir and Reddy [3]	2.4120	1.0960	0.6650	4.7770	3.6880	3.3360
($\epsilon_z = 0$)	Zenkour [17]	2.4141	1.0800	0.6650	4.7879	3.6973	3.3447
	Present	2.4141	1.0980	0.6662	4.7845	3.6958	3.3437
Quasi-3D	Mantari and Canales [21]	-	1.0960	-	-	3.7312	-
($\epsilon_z \neq 0$)	Zenkour [17]	2.4049	1.0966	0.6662	4.8278	3.7628	3.4149
	Present	2.4049	1.0965	0.6661	4.7346	3.6626	3.3147
c. Clamped-clamped beams							
HBT	Khdeir and Reddy [3]	1.5370	0.5320	0.1470	1.9220	1.0050	0.6790
($\epsilon_z = 0$)	Present	1.5378	0.5320	0.1473	1.9227	1.0062	0.6796
Quasi-3D	Mantari and Canales [21]	-	0.5324	-	-	1.0101	-
($\epsilon_z \neq 0$)	Present	1.5487	0.5332	0.1472	1.9193	0.9983	0.6733

Table 3: Normal and shear stresses of [0°/90°/0°] and [0°/90°] simply-supported beams under a uniformly distributed load (MAT 1).

Theory	Reference	Symmetric ([0°/90°/0°])			Anti-symmetric ([0°/90°])		
		$L/h = 5$	10	50	$L/h = 5$	10	50
a. Normal stress ($\overline{\sigma}_x$)							
HBT	Zenkour [17]	1.0669	0.8500	0.7805	0.2362	0.2343	0.2336
($\epsilon_z = 0$)	Present	1.0670	0.8503	0.7809	0.2361	0.2342	0.2336
Quasi-3D	Zenkour [17]	1.0732	0.8506	0.7806	0.2276	0.2246	0.2236
($\epsilon_z \neq 0$)	Mantari and Canales [21]	-	0.8501	-	-	0.2227	-
	Present	1.0670	0.8502	0.7809	0.2428	0.2375	0.2358
b. Shear stress ($\overline{\sigma}_{xz}$)							
HBT	Zenkour [17]	0.4057	0.4311	0.4514	0.9211	0.9572	0.9860
($\epsilon_z = 0$)	Present	0.4057	0.4311	0.4518	0.9187	0.9484	0.8445
Quasi-3D	Zenkour [17]	0.4013	0.4289	0.4509	0.9038	0.9469	0.9814
($\epsilon_z \neq 0$)	Mantari and Canales [21]	-	-	-	-	0.9503	-
	Present	0.4017	0.4295	0.4518	0.9117	0.9474	0.8481

Table 4: Mid-span displacements of $[0°/\theta/0°]$ and $[0°/\theta]$ beams under a uniformly distributed load.

L/h	Lay-ups	0°	15°	30°	45°	60°	75°	90°
a. Cantilever beams								
5	$[0°/\theta]$	5.2774	8.0005	11.6830	13.8390	14.8020	15.1080	15.1540
	$[0°/\theta]$*	5.2774	5.7513	8.2970	12.6470	14.6960	15.1070	15.1540
	$[0°/\theta/0°]$	5.2774	5.4898	5.8804	6.2879	6.6029	6.7919	6.8541
	$[0°/\theta/0°]$*	5.2774	5.4539	5.8684	6.2862	6.6028	6.7919	6.8541
10	$[0°/\theta]$	2.9663	5.5712	9.1499	11.1650	12.0020	12.2260	12.2440
	$[0°/\theta]$*	2.9663	3.3073	5.6046	9.8908	11.8840	12.2250	12.2440
	$[0°/\theta/0°]$	2.9663	3.0653	3.1828	3.2992	3.3889	3.4428	3.4605
	$[0°/\theta/0°]$*	2.9663	3.0272	3.1706	3.2976	3.3888	3.4428	3.4605
50	$[0°/\theta]$	2.1602	4.7429	8.2916	10.2600	11.0540	11.2500	11.2580
	$[0°/\theta]$*	2.1602	2.4579	4.6816	8.9536	10.9310	11.2490	11.2580
	$[0°/\theta/0°]$	2.1602	2.2228	2.2405	2.2483	2.2531	2.2557	2.2565
	$[0°/\theta/0°]$*	2.1602	2.1796	2.2264	2.2464	2.2529	2.2557	2.2565
b. Simply-supported beams								
5	$[0°/\theta]$	1.7930	2.5763	3.6634	4.3135	4.6135	4.7162	4.7346
	$[0°/\theta]$*	1.7930	1.9397	2.6920	3.9683	4.5821	4.7160	4.7346
	$[0°/\theta/0°]$	1.7930	1.8626	2.0140	2.1762	2.3030	2.3796	2.4049
	$[0°/\theta/0°]$*	1.7930	1.8591	2.0132	2.1761	2.3030	2.3796	2.4049
10	$[0°/\theta]$	0.9222	1.6861	2.7403	3.3370	3.5871	3.6562	3.6626
	$[0°/\theta]$*	0.9222	1.0240	1.7012	2.9619	3.5519	3.6558	3.6626
	$[0°/\theta/0°]$	0.9222	0.9529	0.9946	1.0370	1.0700	1.0900	1.0965
	$[0°/\theta/0°]$*	0.9222	0.9429	0.9915	1.0366	1.0700	1.0900	1.0965
50	$[0°/\theta]$	0.6370	1.3966	2.4406	3.0200	3.2540	3.3121	3.3147
	$[0°/\theta]$*	0.6370	0.7245	1.3787	2.6352	3.2176	3.3118	3.3147
	$[0°/\theta/0°]$	0.6370	0.6554	0.6608	0.6634	0.6650	0.6658	0.6661
	$[0°/\theta/0°]$*	0.6370	0.6427	0.6567	0.6628	0.6649	0.6658	0.6661
c. Clamped-clamped beams								
5	$[0°/\theta]$	1.0998	1.3165	1.5755	1.7547	1.8575	1.9060	1.9193
	$[0°/\theta]$*	1.0998	1.1741	1.4172	1.7050	1.8536	1.9059	1.9193
	$[0°/\theta/0°]$	1.0998	1.1537	1.2670	1.3856	1.4766	1.5309	1.5487
	$[0°/\theta/0°]$*	1.0998	1.1490	1.2649	1.3852	1.4765	1.5309	1.5487
10	$[0°/\theta]$	0.3968	0.5584	0.7783	0.9107	0.9726	0.9943	0.9983
	$[0°/\theta]$*	0.3968	0.4286	0.5849	0.8425	0.9665	0.9942	0.9983
	$[0°/\theta/0°]$	0.3968	0.4130	0.4469	0.4828	0.5108	0.5277	0.5332
	$[0°/\theta/0°]$*	0.3968	0.4116	0.4464	0.4828	0.5108	0.5277	0.5332
50	$[0°/\theta]$	0.1367	0.2887	0.4974	0.6137	0.6608	0.6727	0.6733
	$[0°/\theta]$*	0.1367	0.1547	0.2861	0.5372	0.6537	0.6726	0.6733
	$[0°/\theta/0°]$	0.1367	0.1408	0.1431	0.1449	0.1462	0.1470	0.1472
	$[0°/\theta/0°]$*	0.1367	0.1383	0.1423	0.1448	0.1462	0.1470	0.1472

*: γ_{xy} is neglected

Table 5: Normal and shear stresses of $[0°/\theta/0°]$ and $[0°/\theta]$ simply-supported beams under a uniformly distributed load (MAT 1).

L/h	Lay-ups	0°	15°	30°	45°	60°	75°	90°
a. Normal stress ($\overline{\sigma}_x$)								
5	$[0°/\theta]$	0.9498	0.5724	0.3746	0.2852	0.2510	0.2429	0.2428
	$[0°/\theta]$*	0.9498	0.8623	0.5781	0.3429	0.2561	0.2430	0.2428
	$[0°/\theta/0°]$	0.9498	0.9731	1.0010	1.0280	1.0500	1.0630	1.0670
	$[0°/\theta/0°]$*	0.9498	0.9648	0.9986	1.0280	1.0500	1.0630	1.0670
10	$[0°/\theta]$	0.8002	0.5415	0.3661	0.2802	0.2464	0.2379	0.2375
	$[0°/\theta]$*	0.8002	0.7443	0.5451	0.3368	0.2517	0.2380	0.2375
	$[0°/\theta/0°]$	0.8002	0.8222	0.8326	0.8403	0.8459	0.8491	0.8502
	$[0°/\theta/0°]$*	0.8002	0.8086	0.8284	0.8398	0.8458	0.8491	0.8502
50	$[0°/\theta]$	0.7523	0.5315	0.3633	0.2785	0.2449	0.2363	0.2358
	$[0°/\theta]$*	0.7523	0.7066	0.5345	0.3348	0.2502	0.2364	0.2358
	$[0°/\theta/0°]$	0.7523	0.7739	0.7788	0.7801	0.7806	0.7808	0.7809
	$[0°/\theta/0°]$*	0.7523	0.7587	0.7739	0.7795	0.7806	0.7808	0.7809
b. Shear stress ($\overline{\sigma}_{xz}$)								
5	$[0°/\theta]$	0.6679	0.7050	0.7598	0.8208	0.8703	0.9012	0.9117
	$[0°/\theta]$*	0.6679	0.7024	0.7759	0.8283	0.8710	0.9012	0.9117
	$[0°/\theta/0°]$	0.6679	0.6395	0.5729	0.5016	0.4462	0.4128	0.4017
	$[0°/\theta/0°]$*	0.6679	0.6392	0.5727	0.5016	0.4461	0.4128	0.4017
10	$[0°/\theta]$	0.7100	0.7394	0.7913	0.8528	0.9039	0.9363	0.9474
	$[0°/\theta]$*	0.7100	0.7451	0.8157	0.8632	0.9049	0.9363	0.9474
	$[0°/\theta/0°]$	0.7100	0.6783	0.6088	0.5344	0.4762	0.4411	0.4295
	$[0°/\theta/0°]$*	0.7100	0.6798	0.6091	0.5344	0.4762	0.4411	0.4295
50	$[0°/\theta]$	0.7434	0.7443	0.7434	0.7718	0.8085	0.8372	0.8481
	$[0°/\theta]$*	0.7434	0.7782	0.8247	0.8046	0.8117	0.8372	0.8481
	$[0°/\theta/0°]$	0.7434	0.7090	0.6373	0.5605	0.5003	0.4638	0.4518
	$[0°/\theta/0°]$*	0.7434	0.7119	0.6381	0.5606	0.5003	0.4638	0.4518

*: γ_{xy} is neglected

Table 6: Mid-span displacements of $[0^\circ/90^\circ/0^\circ]$ ($h_2/h_1 = 3$ and $h_2/h_1 = 8$) sandwich beams under a uniformly distributed load (MAT 2).

Theory	Reference	$h_2/h_1 = 3$			$h_2/h_1 = 8$		
		$L/h = 5$	10	50	$L/h = 5$	10	50
a. Cantilever beams							
HBT ($\epsilon_z = 0$)	Present	3.8148	2.9717	2.6927	5.1619	4.4281	4.1892
Quasi-3D ($\epsilon_z \neq 0$)	Present	3.8188	2.9704	2.6923	5.1577	4.4250	4.1882
b. Simply-supported beams							
HBT ($\epsilon_z = 0$)	Zenkour [17]	1.1853	0.8879	0.7925	1.5661	1.3135	1.2325
	Present	1.1853	0.8879	0.7925	1.5661	1.3135	1.2325
Quasi-3D ($\epsilon_z \neq 0$)	Zenkour [17]	1.1751	0.8863	0.7924	1.5538	1.3114	1.2325
	Present	1.1751	0.8863	0.7924	1.5538	1.3114	1.2325
c. Clamped-clamped beams							
HBT ($\epsilon_z = 0$)	Present	0.5257	0.2534	0.1616	0.5257	0.2534	0.1616
Quasi-3D ($\epsilon_z \neq 0$)	Present	0.5257	0.2533	0.1616	0.5257	0.2533	0.1616

Table 7: Normal and shear stresses of $[0^\circ/90^\circ/0^\circ]$ ($h_2/h_1=3$ and $h_2/h_1=8$) simply-supported beams under a uniformly distributed load (MAT 2).

Theory	Reference	$h_2/h_1=3$			$h_2/h_1=8$		
		$L/h=5$	10	50	$L/h=5$	10	50
a. Normal stress ($\overline{\sigma}_x$)							
HBT ($\epsilon_z=0$)	Zenkour [17]	0.9980	0.9592	0.9467	1.5044	1.4823	1.4753
	Present	0.9984	0.9596	0.9471	1.5050	1.4830	1.4760
Quasi-3D ($\epsilon_z\neq 0$)	Zenkour [17]	1.0027	0.9603	0.9468	1.5140	1.4850	1.4754
	Present	0.9961	0.9590	0.9497	1.4990	1.4810	1.4760
b. Shear stress ($\overline{\sigma}_{xz}$)							
HBT ($\epsilon_z=0$)	Zenkour [17]	0.7495	0.7641	0.7755	0.6779	0.6852	0.6906
	Present	0.7495	0.7644	0.7771	0.6781	0.6860	0.6922
Quasi-3D ($\epsilon_z\neq 0$)	Zenkour [17]	0.7309	0.7548	0.7740	0.6633	0.6780	0.6897
	Present	0.7373	0.7619	0.7713	0.6735	0.6846	0.6917

Table 8: Displacements and stresses of [0°/90°/C/90°/0°] simply-supported beams under a uniformly distributed load (MAT 3, $\overline{u} = \frac{u(0,h/2)E_2}{hq}$, $\overline{\sigma}_{xz} = \frac{\sigma_{xz}(0,0)}{q}$, $\overline{\sigma}_x = \frac{\sigma_x(L/2,h/2)}{q}$).

L/h	$\overline{w}$		$\overline{u}$		$\overline{\sigma}_{xz}$		$\overline{\sigma}_x$	
	Ref. [28]	Present	Ref. [28]	Present	Ref. [28]	Present	Ref. [28]	Present
5	9.824	9.454	5.053	5.113	2.654	2.788	69.820	71.490
10	3.791	3.730	31.010	31.250	5.641	6.083	225.773	227.100
20	2.242	2.233	227.600	228.700	11.617	12.670	847.181	849.600
50	1.806	1.809	3463.000	3476.000	29.556	32.450	5198.722	5207.000

Table 9: Displacements and stresses of [0°/90°/C/90°/0°] clamped-clamped and cantilever beams under a uniformly distributed load (MAT 3, $\overline{\sigma}_{xz} = \frac{\sigma_{xz}(0,0)}{q}$, $\overline{\sigma}_x = \frac{\sigma_x(L/2,h/2)}{q}$).

L/h	Clamped-clamped beams			Cantilever beams		
	$\overline{w}$	$\overline{\sigma}_{xz}$	$\overline{\sigma}_x$	$\overline{w}$	$\overline{\sigma}_{xz}$	$\overline{\sigma}_x$
5	6.045	0.271	36.770	25.362	0.513	32.460
10	2.070	0.304	88.960	11.342	0.344	187.700
20	0.814	0.127	296.600	7.317	0.485	809.600
50	0.424	0.368	1750.000	6.114	2.802	5163.000

Table 10: Mid-span displacements of [0°/Core/0°] sandwich beams under a uniformly distributed load (MAT 4 and MAT 5).

Reference	MAT 4				MAT 5			
	L/h=4	5	10	20	L/h=4	5	10	20
a. Cantilever beams								
Present	34.2960	2.4034	9.5253	5.6716	30.8400	22.1110	9.9764	6.8098
Carrera et al. [29] (ED2)	35.3330	-	9.6870	5.7068	-	-	-	-
b. Simply-supported beams								
Present	12.4380	8.4671	3.0906	1.7317	10.7560	7.5196	3.1534	2.0528
Pawar et al. [20] (Quasi-3D)	-	-	3.0905	1.7318	-	7.5196	3.1534	-
Kant et al. [19]	13.7505	-	3.3300	1.7935	-	-	-	-
Carrera et al. [29] (ED2)	13.1730	-	3.1448	1.7430	-	-	-	-
c. Clamped-clamped beams								
Present	9.5444	6.4653	1.9385	0.6926	8.2816	5.5851	1.7275	0.6941
Carrera et al. [29] (ED2)	9.9008	-	1.9950	0.7033	-	-	-	-

Table 11: Normal and shear stresses of [0°/Core/0°] simply-supported sandwich beams under a uniformly distributed load (MAT 4 and 5).

Reference	$\overline{\sigma}_x$			$\overline{\sigma}_{xz}$		
	L/h=4	5	10	L/h=4	5	10
a. Sandwich 1 (MAT 4)						
Present	2.4390	2.1140	1.6800	0.5453	0.5572	0.5808
Kant et al. [19]	2.6032	-	1.7290	0.5703	-	0.5240
b. Sandwich 2 (MAT 5)						
Present	2.0970	1.8960	1.6270	0.5916	0.6022	0.6236
Pawar et al. [20]	-	1.8896	1.6309	-	0.5090	0.5312